Consider the integral $\int \frac{\tan^3 2x}{\cos^4 2x}\,dx$. There are two basic approaches to this: (1)convert the cos function to a sec function and work with tan and sec or (2) convert tan to sin and cos and work with sin and cos. Let's work them both and compare:

(1) $\int \frac{\tan^3 2x}{\cos^4 2x}\,dx = \int \tan^3 2x \sec^4 2x\,dx = \int \tan^3 2x\left(1+\tan^2 2x\right)\sec^2 2x\,dx$

$= \int \left(\tan^3 2x + \tan^5 2x\right)\sec^2 2x\,dx$

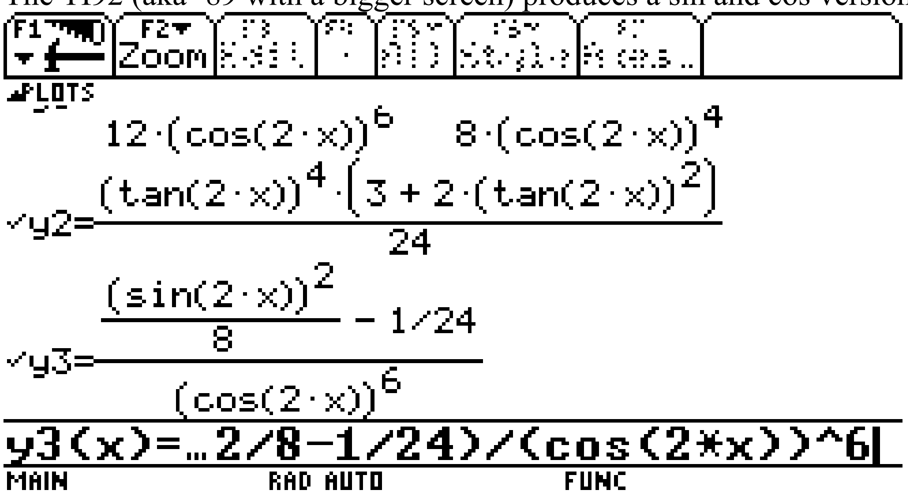
Substitute $u = \tan 2x$ and $du = 2\sec^2 2x dx$ and the integral becomes

$= \frac{1}{2}\int \left(u^3 + u^5\right)du = \frac{u^4}{8} + \frac{u^6}{12} + c = \frac{\tan^4 2x}{24}\left(3 + 2\tan^2 2x\right) + c$

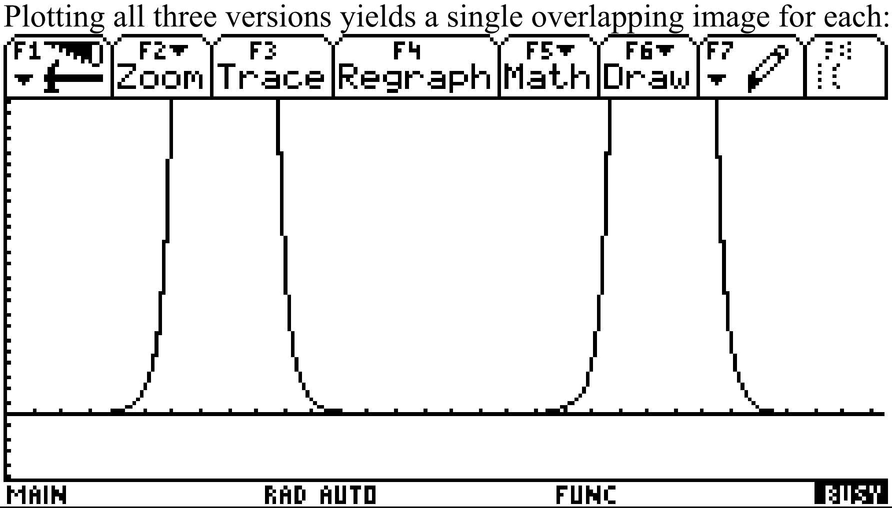
(2) $\int \frac{\tan^3 2x}{\cos^4 2x}\,dx = \int \frac{\sin^3 2x}{\cos^7 2x}\,dx = \int \frac{1-\cos^2 2x}{\cos^7 2x}\sin 2x dx$ Substitute $u = \cos 2x$ and $du = -2\sin 2x\,dx$ and you get

$-\frac{1}{2}\int \frac{1-u^2}{u^7}\,du = -\frac{1}{2}\left(-\frac{1}{6u^6} + \frac{1}{4u^4}\right) + c = \frac{\sec^4 2x}{24}\left(2\sec^2 2x - 3\right) + c$

The TI92 (aka '89 with a bigger screen) produces a sin and cos version:



Plotting all three versions yields a single overlapping image for each:



How about a rational function for a second integral? $\int \frac{5x^2+4x-5}{x^3-x}\,dx$

For this we been the partial fractions expansion of the integrand:

$$\frac{5x^2+4x-5}{x^3-x}=\frac{5x^2+4x-5}{x(x-1)(x+1)}=\frac{A}{x}+\frac{B}{x-1}+\frac{C}{x+1}$$

Clearing denominators leads to

$$5x^2+4x-5=A(x-1)(x+1)+Bx(x+1)+Cx(x-1)$$

At this stage you could expand and equate coefficients of like powers of x on the left and the right, and sometimes this is needed (more irreducible quadratics involved, say) but here it's simpler to observe that the equation must be true for all x so it must be true for $x = 0$: $-5=-A$; for $x = 1$: $4=2B$; and for $x = -1$: $-4=2C$ so $A = 5$, $B = 2$ and $C = -2$. Thus,

$$\int \frac{5x^2+4x-5}{x^3-x}\,dx=\int \frac{5}{x}+\frac{2}{x-1}-\frac{2}{x+1}\,dx=5\ln|x|+2\ln|x-1|-2\ln|x+1|=\ln\left|\frac{x^5(x-1)^2}{(x+1)^2}\right|$$

Oh, professor? Plus a constant.

One more for the road? What if we tweak that denominator in the last problem and make the subtraction an addition? $\int \frac{5x^2+4x-5}{x^3+x}\,dx$ Then the partial fractions form involves an irreducible quadratic: $\frac{5x^2+4x-5}{x^3+x}=\frac{5x^2+4x-5}{x(x^2+1)}=\frac{A}{x}+\frac{Bx+C}{x^2+1}$

Clearing denominators leads to

$$5x^2+4x-5=A(x^2+1)+(Bx+C)x=(A+B)x^2+Cx+A$$

Equating coefficients shows immediately (with more complicated problems this can involve solving a system of linear equations) that $A = -5$, $B = 10$ and $C = 4$. So the integral comes out, as all these rational functions do, as a combo of logs and arctans:

$$\int \frac{5x^2+4x-5}{x^3+x}\,dx=\int \frac{-5}{x}+\frac{10x+4}{x^2+1}\,dx=-5\ln|x|+5\int \frac{2x}{x^2+1}\,dx+4\int \frac{1}{x^2+1}\,dx$$

$$=-5\ln|x|+5\ln|x^2+1|+4\arctan x+c$$

More complicated rational integrals require completing the square of the irreducible quadratic and this can get pretty tricky. Look at the screen shots below, for instance: